LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – MATHEMATICS

FIRST SEMESTER – NOVEMBER 2007

MT 1807 - DIFFERENTIAL GEOMETRY

AB 22

Date : 02/11/2007 Dept. No. Max. : 100 Marks

Time : 1:00 - 4:00

Answer ALL the questions

I a) Show that the principal normal at consecutive points do not intersect unless $\tau = 0$.

(or)

b) Obtain the equation of the tangent at any point of the space curve. [5]

c) Derive the Serret-Frenet formulae for the space curve in terms of Darboux vector[15]

(or)

d) Define osculating plane. Derive the equation of the osculating plane at a point on the curve and also in terms of parameter u.

II a) Find the plane that has three point of contact at origin with the curve $x = u^4 - 1,\ y = u^3 - 1$ and $z = u^2 - 1$.

(or)

b) Find the torsion of the curve $x = a\cos 2u, y = a\sin 2u, z = 2a\sin u$. [5]

c) Derive the equation of evolute and involute. . [15]

(or)

d) State and prove the fundamental theorem for space curve.

III a) Obtain the geometrical interpretation of metric.

(or)

b) Find the first fundamental magnitudes for the curve $\overline{r} = (u\cos v, u\sin v, cv)$. [5]

c) Derive tangential and polar developable associated with a space curve. [15]

(or)

d) (1) Define ordinary point.
(2) Define singular point.
(3) Define tangent plane and normal plane in terms of parameters.
(4) Find the angle between intersecting curves on the surface with reference to parametric curve. [2+2+4+7]

IV a) Mention the duality between space curve and developable.

(or)

b) Prove that the second fundamental form at any point of the surface has the value which equals twice the length of the perpendicular from continuous point to a point on the tangent plane. [5]

c) (1) Derive the equation satisfying principal curvature at point on a surface.
(2) Show that the curves $u + v$ constant , are geodesics on a surface with metric $(1+u^2)du^2 - 2uvdudv + (1+v^2)dv^2$. [7+8]

(or)

d) State and prove Euler's theorem on normal curvature. Also define Gaussian curvature. [15]

V a) Prove that sphere is the only surface in which all points are umbilics.

(or)

b) Derive the Weingarten equation. [5]

c) Derive the partial differential equation of surface theory. [15]

(or)

d) State the fundamental theorem of Surface Theory and demonstrate.

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